Reply to Comment on "Scaling of the quasiparticle spectrum for d-wave superconductors"

Volovik and Kopnin [1] have raised two separate issues in the preceding comment – (1) that the linearization of the quasiparticle spectrum around the gap node limits the maximum temperature for which the discussion in Ref. [2] is valid, and (2) that the use of the geometric average light cone velocity in Ref. [2] gives an incorrect estimate of the crossover scale. Both of these objections are valid and will be discussed below. We note that, with the exception of this incorrectly predicted crossover scale, the results of our paper remain unchanged.

We first address the issue of linearization of the quasiparticle spectrum. As mentioned in [2] the validity of the linearized dispersion is expected to be restricted to a temperature range $T \ll \Delta^2/E_{\rm F}$ (with $\Delta \sim T_c$ is the maximum gap) where excitation energies are small enough such that the quadratic part of the Hamiltonian (the part representing curvature of the Fermi surface) is much smaller than the leading linearized piece. On the other hand, the form of the quasiparticle spectrum described by Volovik and Kopnin (Eq. 1 of Ref. [1]) accounts for the curvature of the Fermi surface and therefore can be used to describing excitations at energy scales up to the order of $\Delta^2/E_{\rm F}$ for quasiclassical calculations for which $\vec{p}$ is considered to be a good quantum number [3,4].

It should be noted however that as mentioned in [2], in practice, the Fermi surface in the high T_c compounds can be quite flat at the gap nodes such that the Fermi surface curvature is smaller than one would expect for a model circular Fermi surface, and thus the range of validity of the linearization used in [2] may be somewhat larger than otherwise expected. It should also be noted that when curvature of the Fermi surface is important (such as for the calculation of the thermal Hall coefficient), it can be effectively treated using perturbation theory [2].

We now turn to the separate issue of the crossover scale. Volovik and Kopnin point out two crossover scales $T_1 \sim T_c\sqrt{H/H_{c2}}$ and $T_2 \sim (T_c^2/E_{\rm F})\sqrt{H/H_{c2}}$. In the absence of magnetic field, the velocity in the direction perpendicular to the Fermi surface is the Fermi velocity $v_{\rm F}$ whereas the velocity tangential to the Fermi surface is roughly $\sim v_{\rm F}T_c/E_{\rm F}$. In a magnetic field, the periodicity of the vortex lattice is given by the magnetic length l_0, thus the Brillouin zone edge is at momentum $k_{max} \sim 1/l_0$. Neglecting the vector potential and assuming that momentum remains a good quantum number, we find that the energies of the states at the zone edge in the two different directions correspond to the two crossover energy scales discussed above. In Ref. [2] it was incorrectly assumed that in a magnetic field, the semiclassical states precess thus obtaining a single geometrically averaged velocity between the two different directions. However, as correctly treated in [4], the Eilenberger semiclassical approach leaves $\vec{p}$ a good quantum number even in a magnetic field so that the momentum states do not actually precess. This result can also be seen from the dynamics of the linearized Hamiltonian in [2].

In fact, however, the situation is somewhat more complicated than the above paragraph would lead us to believe. When we add a magnetic field [2], one component of the vector potential (times $v_{\rm F}$) acts as a periodic scalar potential for delocalized quasiparticles, resulting in a gap at the zone edge of size T_1 in *both* direction in the Brillioun zone. Whether a gap is actually observed in the density of states depends on the details of the band structure. Nonetheless, it is clear that this should be an important crossover scale being that this is also the typical energy of the periodic potential.

The prediction of the additional crossover scale at energy T_2 is due to the somewhat different physics of the bound vortex core states. Volovik and Kopnin [1,3,4] have calculated that the spacing of the core states is approximately T_2 at low energy. In this low energy range, however, the major contribution to the density of states is from the delocalized quasiparticles [3], so we would probably only see this discretization clearly if a gap occurs in the spectrum of the extended states. Thus, an important direction for future research will be to attempt an exact quantum mechanical treatment of the spectrum at these low energies.

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